

Statefinder Parameter for Varying G in Three Fluid System

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Abstract

In this work, we have considered variable G in flat FRW universe filled with the mixture of dark energy, dark matter and radiation. If there is no interaction between the three fluids, the deceleration parameter and statefinder parameters have been calculated in terms of dimensionless density parameters which can be fixed by observational data. Also the interaction between three fluids has been analyzed due to constant G . The statefinder parameters also calculated in two cases: pressure is constant and pressure is variable.

Keywords

1 Introduction

The present acceleration of the universe as favored by the Supernovae type Ia data can be explained by some exotic matter dominated in the present universe which violates the strong energy condition is termed as dark energy Perlmutter et al (1998); Riess et al (1998). This dark energy has the property that it has positive energy and sufficient negative pressure R. R. Caldwell (2002); R. R. Caldwell et al (2002). Dark energy occupies about 73% of the energy of our Universe, while dark matter about 23%

and the usual baryonic matter 4%. There are different candidates obey the property of dark energy given by – quintessence Peebles et al (2002); Caldwell et al (1998), k-essence Armendariz et al (2000), tachyon A. Sen (2002), phantom R. R. Caldwell (2002), ghost condensate Arkani et al (2004); Piazza et al (2004), quintom Feng et al (2005); Guo et al (2005), brane world models Sahni, Shtanov (2003) and Chaplygin gas models Kamenshchik et al (2001).

Einstein's field equations have two parameters – the Newton's gravitational constant and the cosmological constant. The Newton's gravitational constant G plays the role of a coupling between geometry and matter in the Einstein field equations. In an evolving universe, it appears natural to look at this “constant” as a function of time. Dirac P. A. M. Dirac (1937) proposed for the first time the idea of a variable G on certain physical grounds. He has shown that $G \propto t^{-1}$, but his model ran in some difficulties. Some authors A-M. M. Abdel Rahaman (1990); C. Mass (1994) have shown that G is an increasing function of time. Many other extensions of Einstein's theory with time dependent G have also been proposed in order to achieve a possible unification of gravitation and elementary particle physics or to incorporate Mach's principle in general relativity Hoyle, Narlikar (1964, 1971); Brans, Dicke (1961). Canuto and Narlikar Canuto, Narlikar (1980) have shown that the G -varying cosmology is consistent with whatever cosmological observations presently available. According to Dirac's large numbers hypothesis, $\dot{G}/G \sim$ Hubble rate H V. N. Melnikov (2009). Observations of Hulse-Taylor binary pulsar B1913+16 gives the estimate $0 < \dot{G}/G \sim 2 \pm 4 \times 10^{-12} yr^{-1}$ G. S. B. Kogan (2006) and helioseismological data gives the bound $0 < \dot{G}/G \sim 1.6 \times 10^{-12} yr^{-1}$ D. B. Guenther (1998); S. Ray, U. Mukhopadhyay (2007). Several works on variable G have been studied in last few decades

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M. Jamil, U. Debnath (2011); K.D. Krori et al (2000); M. Jamil (2010); A. I. Arbab (1998, 2001, 2002, 2008).

In this work, we have considered the universe is filled with radiation, dark matter and dark energy with and without interactions. We now follow the method of the ref. M. Jamil (2010) in three fluid system. The dimensionless density parameters and statefinder parameters have been calculated due to variable G in three fluid system of flat FRW universe.

2 Three Fluids with Varying Gravitational Constant

The isotropic, homogeneous and flat FRW model of the universe is described by the line element

$$ds^2 = dt^2 - a^2(t) [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \quad (1)$$

where $a(t)$ is the scale factor. The energy-momentum tensor is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \quad (2)$$

where u_μ is the four velocity satisfying $u^\mu u_\mu = 1$. Here ρ and p are respectively the energy density and pressure of the fluid in the universe.

The Einstein's field equations are given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(t)T_{\mu\nu}, \quad (3)$$

where $R_{\mu\nu}$, $g_{\mu\nu}$ and R are Ricci tensor, metric tensor and Ricci scalar respectively. Here we consider gravitational constant G as a function of cosmic time t . Now assume that the universe is filled with the mixture of three fluids (radiation, dark matter and dark energy), so from the equations (1), (2) and (3) we have the Einstein's field equations as

$$H^2 = \frac{8\pi G(t)}{3}\rho, \quad (4)$$

and

$$\dot{H} = -4\pi G(t)(\rho + p). \quad (5)$$

Also the conservation equation is given by

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (6)$$

where, $\rho = \rho_x + \rho_m + \rho_r$ and $p = p_x + p_m + p_r$ are the total energy density and pressure of the fluid in the universe. Here, ρ_x , ρ_m and ρ_r are the energy densities of the dark energy, dark matter and radiation fluids respectively. For dark matter, we choose negligible

pressure i.e., $p_m = 0$. Also p_x and p_r are the pressures of dark energy and radiation respectively. Now the equation of state for radiation is given as $p_r = \frac{1}{3}\rho_r$. Now assume that the equation of state for dark energy is $p_x = w\rho_x$. Next we study the non-interacting and interacting situations.

3 Non-Interacting case

If there is no interaction between dark energy, dark matter and radiation then from equation (6), we can write the continuity equations for dark matter, dark energy and radiation as

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (7)$$

$$\dot{\rho}_x + 3H(1 + w)\rho_x = 0, \quad (8)$$

$$\dot{\rho}_r + 4H\rho_r = 0. \quad (9)$$

Now we define the dimensionless density parameters in the form

$$\Omega = \frac{\rho}{\rho_{cr}}, \quad \Omega_x = \frac{\rho_x}{\rho_{cr}}, \quad \Omega_m = \frac{\rho_m}{\rho_{cr}}, \quad \Omega_r = \frac{\rho_r}{\rho_{cr}} \quad (10)$$

where ρ_{cr} is the critical density, so we obtain

$$\Omega = \Omega_x + \Omega_m + \Omega_r. \quad (11)$$

The deceleration parameter $q = -\frac{\ddot{a}}{aH^2}$ can be expressed as in terms of density parameters in the following

$$q = \frac{1}{2}[(1 + 3w)\Omega_x + \Omega_m + \Omega_r]. \quad (12)$$

From above we obtain

$$\dot{q} = \frac{1}{2}[(1 + 3w)\dot{\Omega}_x + 3\dot{w}\Omega_x + \dot{\Omega}_m + \dot{\Omega}_r]. \quad (13)$$

From (9), we have

$$\dot{\Omega} = \frac{\dot{\rho}}{\rho_{cr}} - \frac{\rho}{\rho_{cr}^2}\dot{\rho}_{cr}, \quad (14)$$

where

$$\dot{\rho}_{cr} = \rho_{cr} \left(2\frac{\dot{H}}{H} - \frac{\dot{G}}{G} \right). \quad (15)$$

Furthermore,

$$\dot{H} = -H^2(1 + q). \quad (16)$$

So from (14) we have

$$\dot{\rho}_{cr} = -H\rho_{cr}(2(1 + q) + \Delta G), \quad (17)$$

where $\Delta_G \equiv G'/G$, $\dot{G} = HG'$. Now from (13) we get

$$\dot{\Omega} = \frac{\dot{\rho}}{\rho_{cr}} + \Omega H [2(1+q) + \Delta G]. \quad (18)$$

From equations (7) to (10), we yield

$$\dot{\Omega}_m = \Omega_m H [-1 + 2q + \Delta G], \quad (19)$$

$$\dot{\Omega}_x = \Omega_x H [-1 - 3w + 2q + \Delta G], \quad (20)$$

and

$$\dot{\Omega}_r = \Omega_r H [-2 + 2q + \Delta G]. \quad (21)$$

Substituting equations (19) - (21) in equation (13) and after simplifying, we get

$$\dot{q} = \frac{1}{2} \Omega_m H (-1 + 2q + \Delta G) + \Omega_x H \left\{ (1 + 3w) + \frac{3}{2} \dot{w} \right\} (-1 - 3w + 2q + \Delta G) + \Omega_r H (-2 + 2q + \Delta G) \quad (22)$$

We also determine the dimensionless pair of cosmological diagnostic pair $\{r, s\}$ dubbed as statefinder parameters introduced by Sahni et al (2003). The two parameters have a great geometrical significance since they are derived from the cosmic scale factor alone. Also this pair generalizes the well known geometrical parameters like the Hubble parameter and the deceleration parameter. The parameter r forms the next step in the hierarchy of geometrical cosmological parameters H and q .

The diagnostic pair has been used for holographic dark energy model Setare et al (2007); Jamil et al (2009); Setare & Vagenas (2009); Setare (2006); Setare & Jamil (2011), Chaplygin gas Jamil & Debnath (2011); Debnath & Jamil (2011).

The parameters are given by

$$r = \frac{\ddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r - 1}{3(q - 1/2)}. \quad (23)$$

For varying gravitational constant G , the definition of s can be generalized to M. Jamil (2010)

$$s = \frac{r - \Omega}{3(q - \frac{\Omega}{2})}. \quad (24)$$

From (23), we can write r in terms of H and q as

$$r = 2q^2 + q - \frac{\dot{q}}{H}. \quad (25)$$

Using (12) and (22), we can write equation (25) in the form

$$\begin{aligned} r = & \frac{1}{2} [(1 + 3w) \Omega_x + \Omega_m + \Omega_r]^2 \\ & + \frac{1}{2} [(1 + 3w) \Omega_x + \Omega_m + \Omega_r] \\ & - \frac{1}{2} \Omega_m (-1 + 2q + \Delta G) \\ & + \Omega_x \left\{ (1 + 3w) + \frac{3}{2} \dot{w} \right\} \times \\ & (-1 - 3w + 2q + \Delta G) - \Omega_r (-2 + 2q + \Delta G) \end{aligned} \quad (26)$$

Substituting this in equation (24), we get

$$\begin{aligned} s = & \frac{1}{3w\Omega_x} \{ [(1 + 3w) \Omega_x + \Omega_m + \Omega_r]^2 \\ & + [(-1 + 3w) \Omega_x + \Omega_r] - \Omega_m (2q + \Delta G) \\ & + 2\Omega_x \left\{ (1 + 3w) + \frac{3}{2} \dot{w} \right\} \times (-1 - 3w + 2q + \Delta G) \\ & - 2\Omega_r (-1 + 2q + \Delta G) \} \end{aligned} \quad (27)$$

4 Interacting Case

Interacting models where the dark energy weakly interacts with the dark matter have also been studied to explain the evolution of the Universe. These models describe an energy flow between the components. To obtain a suitable evolution of the Universe an interaction is often assumed such that the decay rate should be proportional to the present value of the Hubble parameter (Jamil & Debnath (2011)) for good fit to the expansion history of the Universe as determined by the Supernovae and CMB data M. S. Berger (2006). These kind of models describe an energy flow between the components so that no components are conserved separately. First, we assume that the dark matter component is interacting with dark energy component, so the continuity equations of dark matter and dark energy are

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (28)$$

$$\dot{\rho}_x + 3H(1+w)\rho_x = -Q', \quad (29)$$

where Q and Q' in order to include the scenario in which the mutual interaction between the two principal components of the universe leads to some loss in other forms of cosmic constituents. In this case, we have assumed $Q \neq Q'$, so from (6), we have the continuity equation

for radiation fluid as Cruz et al (2008); Debnath (2010); Jamil et al (2010); Jamil & Rahaman (2009)

$$\dot{\rho}_r + 4H\rho_r = Q' - Q. \quad (30)$$

If $Q < Q'$, then part of the dark energy density decays into dark matter and the rest in the radiation fluid. But if $Q > Q'$, then dark matter receives energy from dark energy and from radiation. We are taking about in this case that dark energy decay into dark matter (or vice versa, depending on the sign of Q) and radiation. Assume, the interaction terms Q and Q' are $Q = 3\Pi_1 H$ and $Q' = 3\Pi_2 H$ which measure the strength of interactions where Π_1 and Π_2 have the dimension of density. Now assume that G is constant. So Differentiating equation (5), we have

$$\ddot{H} = -4\pi G (\dot{\rho} + \dot{p}). \quad (31)$$

Also the deceleration parameter q can be expressed as

$$q = -\frac{1}{2} - \frac{3}{2} \frac{p}{\rho}. \quad (32)$$

• **Case-I:** when $\dot{p} = 0$. Equation (31) reduces to

$$\begin{aligned} \ddot{H} = & -4\pi G H \left[-3\rho_m - 3\left(1 - \dot{w} - w^2\right)\rho_x \right. \\ & \left. - w \frac{Q'}{H} \rho_x - 4\rho_r \right]. \end{aligned} \quad (33)$$

Dividing by H^2 , we yield

$$\frac{\ddot{H}}{H^3} = 9 - 9 \frac{\left\{ (\dot{w} + w\Pi_2 + w^2) \rho_x + \left(-\frac{7}{3}\right) \rho_r \right\}}{\rho}. \quad (34)$$

Now equation (25) can be written as

$$r = \frac{\ddot{H}}{H^3} - 3q + 2. \quad (35)$$

Using (34), the above equation becomes

$$r = \frac{25}{2} + \frac{9p}{4\rho} - 9 \frac{(\dot{w} + w\Pi_2 + w^2) \rho_x}{\rho}. \quad (36)$$

Also from (23), we obtain the expression for s as

$$s = \frac{\frac{23}{2} + \frac{9p - 36(\dot{w} + w\Pi_2 + w^2) \rho_x}{4\rho}}{3 \left(-1 - \frac{3}{2} \frac{p}{\rho} \right)}. \quad (37)$$

• **Case-II:** when $\dot{p} \neq 0$ then for $p = p_x + p_r = w\rho_x + \frac{1}{3}\rho_r$ we get (from (31))

$$\begin{aligned} \frac{\ddot{H}}{H} = & -4\pi G \left\{ -3\rho_m + \left(-3 - 6w - 3w^2 + \frac{1}{H} \dot{w} \right) \rho_x \right. \\ & \left. - \frac{16}{3} \rho_r - (3w - 1) \Pi_2 - \Pi_1 \right\} \times \\ & \left\{ (1 - \dot{w}) - w \frac{Q'}{H} \rho_x - 4\rho_r \right\}. \end{aligned} \quad (38)$$

Dividing by H^2 we obtain

$$\begin{aligned} \frac{\ddot{H}}{H^3} = & \frac{3}{2} \frac{1}{\rho} \left\{ -3\rho_m + \left(-3 - 6w - 3w^2 + \frac{1}{H} \dot{w} \right) \rho_x \right. \\ & \left. - \frac{16}{3} \rho_r - (3w - 1) \Pi_2 - \Pi_1 \right\} \times \\ & \left\{ (1 - \dot{w}) - w \frac{Q'}{H} \rho_x - 4\rho_r \right\}. \end{aligned} \quad (39)$$

In this case, the expressions of r and s become

$$r = \frac{7}{2} - \frac{9}{2} \frac{\rho_m}{\rho} + \left(\frac{3}{2} D_1 + \frac{9}{2} w \right) \frac{\rho_x}{\rho} - \frac{13}{2} \frac{\rho_r}{\rho} - \frac{3}{2} \frac{D_2}{\rho}, \quad (40)$$

and

$$s = \frac{\frac{5}{2} - \frac{9}{2} \frac{\rho_m}{\rho} + \left(\frac{3}{2} D_1 + \frac{9}{2} w \right) \frac{\rho_x}{\rho} - \frac{13}{2} \frac{\rho_r}{\rho} - \frac{3}{2} \frac{D_2}{\rho}}{3 \left(-1 - \frac{3}{2} \frac{p}{\rho} \right)}, \quad (41)$$

where

$$\begin{aligned} D_1 = & -3 - 6w - 3w^2 + \frac{1}{H} \dot{w}, \\ D_2 = & -(3w - 1) \Pi_2 - \Pi_1. \end{aligned}$$

5 Discussions

In this work, we have considered variable G in flat FRW universe filled with the mixture of dark energy, dark matter and radiation. If there is no interaction between the three fluids, the deceleration parameter and statefinder parameters have been calculated in terms of dimensionless density parameters which can be fixed by observational data. Also the interaction between three fluids has been analyzed due to constant G . If $Q < Q'$, then part of the dark energy density decays into dark matter and the rest in the radiation fluid. But if $Q > Q'$, then dark matter receives energy from dark energy and from radiation. We are taking about in this case that dark energy decay into dark matter (or vice versa, depending on the sign of Q) and radiation. The statefinder parameters also calculated in two cases: pressure is constant and pressure is variable. In the literature, the diagnostic pair has been calculated for the

dark energy interacting with dark matter Jamil et al (2010). The interaction of these two dark fluids with a third component has been studied in literature, and hence motivated by this, we calculated the statefinder parameters for the triple fluid interacting model. This model will be useful for investigating further the triple coincidence problem Arkani et al (2004).

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